Section 3.4 (page 195)

1. $f' > 0, f'' > 0$ **3.** $f' < 0, f'' < 0$

5. Concave upward: $(-\infty, \infty)$

7. Concave upward: $(-\infty, 1)$; Concave downward: $(1, \infty)$

9. Concave upward: $(-\infty, 2)$; Concave downward: $(2, \infty)$

11. Concave upward: $(-\infty, -2), (2, \infty)$
Concave downward: $(-2, 2)$

13. Concave upward: $(-\infty, -1), (1, \infty)$
Concave downward: $(-1, 1)$

15. Concave upward: $(-2, 2)$
Concave downward: $(-\infty, -2), (2, \infty)$

17. Concave upward: $(-\pi/2, 0)$; Concave downward: $(0, \pi/2)$

19. Points of inflection: $(-2, -8), (0, 0)$
Concave upward: $(-\infty, -2), (0, \infty)$
Concave downward: $(-2, 0)$

21. Point of inflection: $(2, 8)$; Concave downward: $(-\infty, 2)$
Concave upward: $(2, \infty)$

23. Points of inflection: $(\pm 2\sqrt{3}/3, -20/9)$
Concave upward: $(-\infty, -2\sqrt{3}/3), (2\sqrt{3}/3, \infty)$
Concave downward: $(-2\sqrt{3}/3, 2\sqrt{3}/3)$

25. Points of inflection: $(2, -16), (4, 0)$
Concave upward: $(-\infty, 2), (4, \infty)$; Concave downward: $(2, 4)$

27. Concave upward: $(-3, \infty)$

29. Points of inflection: $(-\sqrt{3}/3, 3), (\sqrt{3}/3, 3)$
Concave upward: $(-\infty, -\sqrt{3}/3), (\sqrt{3}/3, \infty)$
Concave downward: $(-\sqrt{3}/3, \sqrt{3}/3)$

31. Point of inflection: $(2\pi, 0)$
Concave upward: $(2\pi, 4\pi)$; Concave downward: $(0, 2\pi)$

33. Concave upward: $(0, \pi), (2\pi, 3\pi)$
Concave downward: $(\pi, 2\pi), (3\pi, 4\pi)$

35. Points of inflection: $(\pi, 0), (1.823, 1.452), (4.46, -1.452)$
Concave upward: $(1.823, \pi), (4.46, 2\pi)$
Concave downward: $(0, 1.823), (\pi, 4.46)$

37. Relative minimum: $(5, 0)$ **39.** Relative maximum: $(3, 9)$

41. Relative maximum: $(0, 3)$; Relative minimum: $(2, -1)$

43. Relative minimum: $(3, -25)$

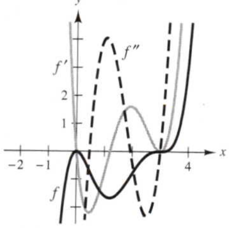
45. Relative maximum: $(2.4, 268.74)$; Relative minimum: $(0, 0)$

47. Relative minimum: $(0, -3)$

49. Relative maximum: $(-2, -4)$; Relative minimum: $(2, 4)$

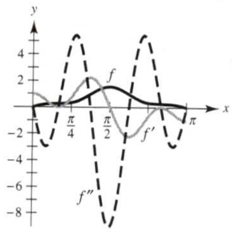
51. No relative extrema, because f is nonincreasing.

53. (a) $f'(x) = 0.2x(x-3)^2(5x-6)$
$f''(x) = 0.4(x-3)(10x^2 - 24x + 9)$
(b) Relative maximum: $(0, 0)$
Relative minimum: $(1.2, -1.6796)$
Points of inflection: $(0.4652, -0.7048)$, $(1.9348, -0.9048)$, $(3, 0)$
(c)



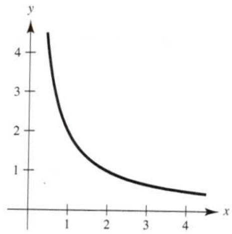
f is increasing when f' is positive, and decreasing when f' is negative. f is concave upward when f'' is positive, and concave downward when f'' is negative.

55. (a) $f'(x) = \cos x - \cos 3x + \cos 5x$
$f''(x) = -\sin x + 3\sin 3x - 5\sin 5x$
(b) Relative maximum: $(\pi/2, 1.53333)$
Points of inflection: $(\pi/6, 0.2667), (1.1731, 0.9637)$, $(1.9685, 0.9637), (5\pi/6, 0.2667)$
(c)

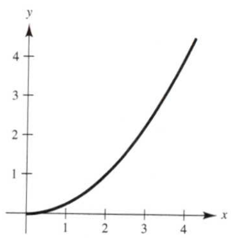


f is increasing when f' is positive, and decreasing when f' is negative. f is concave upward when f'' is positive, and concave downward when f'' is negative.

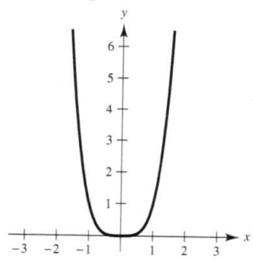
57. (a)



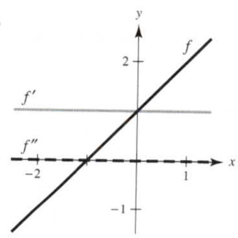
(b)



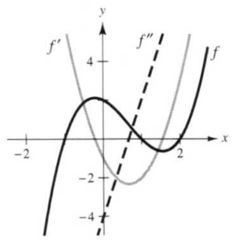
59. Answers will vary. Example: $f(x) = x^4$; $f''(0) = 0$, but $(0, 0)$ is not a point of inflection.



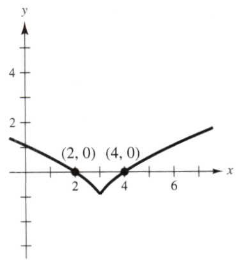
61.



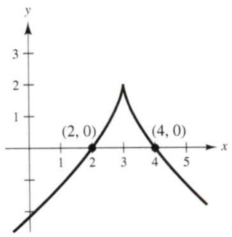
63.



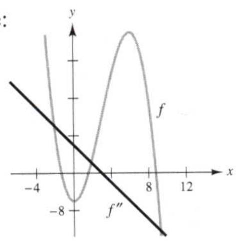
65.



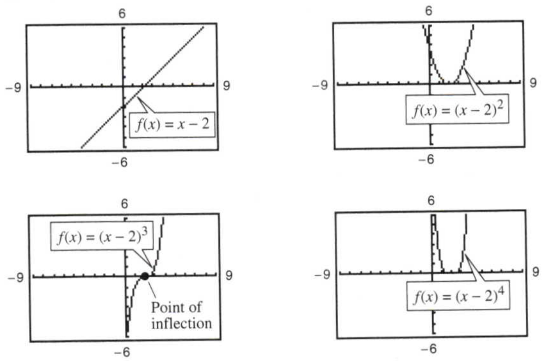
67.



69. Example:



71. (a) $f(x) = (x-2)^n$ has a point of inflection at $(2, 0)$ if n is odd and $n \geq 3$.



(b) Proof

73. $f(x) = \frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x - 24$

75. (a) $f(x) = \frac{1}{32}x^3 + \frac{3}{16}x^2$ (b) Two miles from touchdown

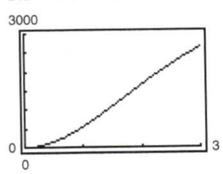
77. $x = \left(\dfrac{15 - \sqrt{33}}{16}\right)L \approx 0.578L$ **79.** $x = 100$ units

81. (a)

t	0.5	1	1.5	2	2.5	3
S	151.5	555.6	1097.6	1666.7	2193.0	2647.1

$1.5 < t < 2$

(b)



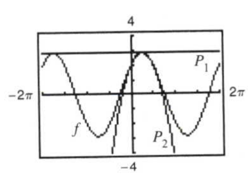
$t \approx 1.5$

(c) About 1.633 yr

83. $P_1(x) = 2\sqrt{2}$

$P_2(x) = 2\sqrt{2} - \sqrt{2}(x - \pi/4)^2$

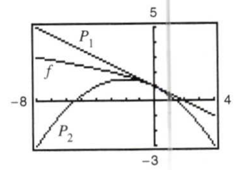
The values of f, P_1, and P_2 and their first derivatives are equal when $x = \pi/4$. The approximations worsen as you move away from $x = \pi/4$.



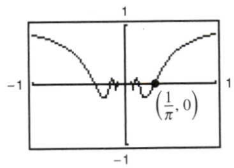
85. $P_1(x) = 1 - x/2$

$P_2(x) = 1 - x/2 - x^2/8$

The values of f, P_1, and P_2 and their first derivatives are equal when $x = 0$. The approximations worsen as you move away from $x = 0$.



87.



89. Proof **91.** True

93. False. f is concave upward at $x = c$ if $f''(c) > 0$. **95.** Proof